Certificate or Subscription? The Optimal Pricing Strategy of Massive Open Online Courses

Abstract

Massive Open Online Courses (MOOCs) are raising worldwide concerns now. The educational characteristic of MOOCs makes it different from the traditional information goods, and its pricing strategy therefore deserves high attention. By observing that major MOOC platforms started to move from the certificate business model to the subscription one, we focus on the business model selection problem faced by MOOC platforms. In this paper, we construct a game theoretical model with a MOOCs platform, an educational institution, and a group of learners. We study the profitability of three models: the certificate model, the subscription model, and the mixed model. We find that learners' bounded rationality is a key for MOOC platforms to introduce the subscription option to learners. Interestingly, while having some learners overestimating their ability benefits the institution and platform, it does not matter whether learners underestimate themselves or not.

Keywords: Massive open online courses, bounded rationality, pricing, game theory.

1、Introduction

(1)、Background and Motivations

The advance of technology has significantly changed our life, especially the way we learned. One of the impact on traditional education is the rising of online education. Massive Open Online Courses (MOOCs), free online courses that are available for anyone to enroll, offer more flexibility to students and greatly reduce the learning cost compared with traditional education. Moreover, the idea of MOOCs allows the contents of courses to reach all over the world as the internet spread, while speeding up the delivery of knowledge.

As of 2017, there have been more than 750 universities that participated in MOOCs (Shah, 2017). The top three MOOC platforms of 2017 are Coursera, edX, and Udacity according to Credentialing, Course Diversity, Course Feature, Social Feature and Partner Institutions (Reviews.com, 2017). Dhawal Shah's *Class Central's Top 50 MOOCs of All Time* (2017 edition) also points out that Coursera is the top provider with 28 courses in the Top 50, and edX is second with 9 courses. Stanford and MIT top the list with four courses each.

The main reasons for educational institutions to offer MOOCs are extending reach and access of knowledge, improving reputation, and ultimately increasing revenues (Hollands and Tirthali, 2014). However, while MOOC platforms also share these objectives, it is critical for MOOC platforms to find a successful business model to be financially sustainable. For MOOC platforms, business model and platform sustainability are the central issues (Baker and Passmore, 2016). When MOOC platforms emerged around 2012, the main business model is to sell course certificates. A learner may study course materials for free. After she completes a course, she may pay for a certificate only if she wants to earn one. This traditional model has been discouraged recently; instead, a new subscription model has been suggested by at least Coursera and Udacity. Under this model, a learner who wants a certificate should first subscribe a course by paying a monthly fee. She then studies the materials in her own pace until the course is completed (or she unsubscribe). A certificate will be awarded to her upon course completion at no additional charge.[1]

Since the profit model of MOOC platforms has held in high regard today, the pricing strategy of MOOC platform also appeal to us. Once a platform adopts a revenue model (free-material-paid-certificate or subscription), educational institutions must follow and determine the prices (certificate prices or monthly subscription fees) accordingly. Different models

[1] Currently on Coursera, only specialization, each containing a series of related courses and a capstone project, adopts the subscription model. It is reasonable to doubt that Coursera is taking an experiment on the two revenue models. If the subscription model can be proved to be financially more beneficial, it will be applied on individual courses ultimately. To focus on the economic incentive issues of the traditional model and the subscription model, we will not consider the difference between individual courses and specialization.

obviously induce institutions choose different prices, which together provide a learner different incentives about course participation and effort exertion. We investigate the profitability of the two revenue models and explain the underlying reasons from the perspective of incentives. For ease of exposition, throughout this study we will call the free-material-paid-certificate model *the certificate model* and the other *the subscription model*.

(2)、Research objectives

As mentioned above, we aim to verify which the free-material-paid-certificate model or the subscription model is more desirable for MOOC platforms. In this study, we will develop a game theoretical model to describe the demand and supply market of MOOCs. There are three players in our model, which are MOOC platforms, educational institutions, and learners. We assume that there is a monopoly platform in the MOOCs market, which connects the MOOCs supply side and demand side, and the main task of the platform is to select the revenue model in either one of the free-material-paid-certificate model and the subscription model, or both. There is an institution offering a MOOC on the platform. As the provider of the MOOC, the institution has the right to determine the price of the course certificate and the monthly subscription fee. Learners heterogeneous in their willingness to pay for a certificate, efficiency in completing the course, and rationality in estimating the course difficulty exist in the market. They decide whether to subscribe to the course, purchase a certificate after completing the course, or paying nothing. We adopt a game-theoretic analysis to solve for the revenue-maximizing pricing strategy and investigate its underlying driving forces.

In this model, both the platform and the institution are looking for revenue maximization, while learners care about their own utility. As the platform and the institution are on the same side, the former will choose the revenue model which is more profitable first, then the later determine the appropriate price depends on the selected model. At the end of the game, leaners make decisions according to the payment and their own ability about the courses. Under this setting, we plan to provide some economic insights about the two revenue model of MOOCs market.

(3)、Research plan

In the next section, we will review some related works about MOOCs and the platform pricing strategies. In Section 3, we will construct a game-theoretic model to display how students react facing different online course payment and the profitability of each collection manner. Preliminary analysis and expected results are then provided in Section 4.

2、Literature Review

(1)、MOOCs

Regarding the reasons for MOOCs to be popular not only among learners but also among universities, several researchers have provided different arguments, which mainly focus on three aspects: learners, educational institutions, and MOOC platforms. For learners, most researchers tend to investigate the motivation of using MOOCs and the factors that affect course completion rate (Hew and Cheung, 2014). For educational institutions, lecture design and the integration of traditional education and online courses raise the most concern (Lawton and Katsomitros, 2012). In this section, we briefly review related works about this issue.

Hew and Cheung (2014) summarize the motivations and challenges of students' and instructors' use of massive open online courses (MOOCs). By processing a total of 362 related articles as of July 31, 2013, they suggest four reasons why students sign up for MOOCs, which are the desire to learn about a new topic or to extend current knowledge, the curious about MOOCs, personal challenging issues, and the desire to collect as many completion certificates as possible.

Hollands and Thirthali (2014) investigate reasons for universities to offer MOOCs. They list six possible reasons, and conclude that extending reach and access and building and maintaining brand are ranked as the top reasons. We can find that even it is still hard to generate profits by offering MOOCs, universities are still willing to do so for educational purpose.

Baker and Passmore (2016) then propose five major value propositions for MOOCs: headhunting, certification, face-to-face learning, personalized learning, and marketing of integration with services external to the MOOC.

As both universities and learners are highly interested in MOOCs, it makes sense for some companies to run platforms to connect these two parties. In the next section, we will review several studies about the revenue models and revenue sharing mechanisms of MOOC platforms.

(2)、MOOC platform pricing strategy and revenue sharing mechanism

Jia et al. (2017) form a theoretical model against the certificate pricing strategy in B2C (business to customer) market, and analyze 1236 real MOOCs certificate selling data. They point out that the best-selling MOOCs are economic and management courses, whereas MOOCs with highest payment rate are those science and engineering courses, and according to law of diminishing returns, the witness to pay of customer for courses offered repeatedly declines.

Regardless which kind of MOOCs make the most profit, since MOOC platforms do not create courses themselves, they need to cooperate with educational institutions. Kolowich (2013)

point out that both EdX and Coursera share part of their revenue from MOOCs for educational institutions. Its purpose is to induce the institution to provide high quality MOOCs. He further indicate that when it comes to profit generate of MOOC platform, the design of revenue sharing mechanism plays an important role.

Back to pricing strategy. For information goods, Sundararajan (2004) prove that comparing to usage-based pricing, the introduction of fixed fee subscription model increase both consumer surplus and total surplus. But when transaction cost exists, it is more profitable to offer both the usage-based price and the monthly subscription fee.

Despite the fact that there have been several researchers provided different arguments on the MOOCs business model and the comparison between usage-based and subscription. Seldom of them discussing the combination of these two objects, and there is still no evidence to show which of the revenue model is more suitable for MOOCs market currently. Hence, we try to investigate above problems and hope to contribute to the MOOCs research field.

3、Model

We consider a MOOC platform (it) connecting an educational institution (she) and a group of learners (for each of them, he). The platform chooses to implement one of the two revenue models, the certificate model and the subscription model, or both (which is called the mixed model). The most significant difference between the two revenue models is that the certificate model allows learners to view the course materials for free and then pay if they want to get a certificate. On the contrary, the subscription model requests learners to pay a monthly subscription fee first. They may then have the right to access the course materials and obtain a certificate after completing the course. The institution sets up a course and determining the certificate price and/or the monthly subscription fee. A learner decides whether to purchase a certificate (if there is a certificate option), subscribe to the course (if there is a subscription option), or does not pay anything. There will of course be some learners viewing the course materials without paying anything.

For whatever revenue generated through certificate sales and/or subscription, the platform and institution share the revenue proportionally. The platform, which is a profit maximizer, needs to consider the institution’s pricing decision when choosing the revenue model. Note that though institutions in general are not profit maximizer (e.g., many of them care more about reputation in practice), when it is about the pricing decision, all an institution may do is to

maximize its revenue.[2] Therefore, to maximize its own profit, the platform should choose the model which allows the institution to maximize her profit (which is equivalent to revenue as there is no variable cost).

A learner will gain a positive utility u after obtaining the certificate. Obviously, different learners evaluate the certificate differently. Therefore, we assume that u spread uniformly in $[0, 1]$. Having a higher value of u means a learner evaluates the certificate more and thus is willing to pay more. Learners are also heterogeneous in their intelligence, background knowledge for the course, and amount of time to invest in this course. To capture this dimension of heterogeneity, we assume that there are *efficient* learners who take t_L months to complete the course and *inefficient* ones taking $t_H > t_L$ months. To avoid tedious derivations with no managerial implication, we normalize t_L to 1 and simply denote t_H by $t > 1$ throughout this study. The proportion of learners who are efficient is $\beta \in (0, 1)$. Let $\tau \in \{1, t\}$ be the amount of time one spends on completing the course.

A learner considers whether to purchase a certificate or subscribe to the course by considering the certificate price p and monthly subscription fee f. Let the total amount of money paid by a learner be T, his utility of obtaining the certificate is $u - T$. Under the certificate model, T is simply the certificate price p, which has nothing to do with one's learning time. A learner's utility of purchasing a certificate is thus $U_C = u - p$. Under the subscription model, we have $T = \tau f$, which depends on the amount of time one spends on completing the course. Therefore, a learner's utility of subscribing the course is $U_S = u - \tau f$. A learner receives a null utility if he does not obtain the certificate.

Naturally, not all learners understand themselves and the course well. In particular, a learner may underestimate or overestimate the time he needs to complete the course. To capture this bounded rationality, we assume that some efficient learners overestimate the learning time and believe that they need t months to complete the course (while they actually need just 1 month). Let the proportion of *irrational* efficient learners by $\alpha_E \in [0,1]$.[3] Similarly, some inefficiency learners underestimate the learning time and believe that they need only 1 month (while they actually need t months). We denote the proportion of inefficient learners that make such a mistake by $\alpha_I \in [0,1]$. Note that if $\alpha_E = \alpha_I = 0$, all learners are fully rational. In this case, all learners perfectly estimate the amount of money to pay and make their decisions accordingly. On the contrary, if any of α_E or α_I is positive, there will be irrational learners.An

[2] This is because under either revenue model, learners have the option of viewing course materials for free. The only difference of the revenue models is on the amount of revenue to generate.

[3] A better way to describe these learners should be "boundedly rational," not "irrational." Nevertheless, to avoid lengthy sentences, we will just use the term "irrational" in the sequel.

irrational learner will estimate his total subscription fee based on his belief on the learning time, not the true learning time. We assume that t is sufficiently small so that an irrational learner will find it optimal to complete the course and earn the certificate even after he finds his true learning time. The case that t is large so that some learners may quit does not bring in additional managerial insights is omitted. To avoid tedious derivations that do not bring in useful insights, we will assume that $t \leq 2$ throughout this study.

The sequence of events is the following. First, the platform chooses one of the three revenue models: the certificate model, subscription model, and mixed model. Second, the institution makes her pricing decision. Under the certificate model, the institution sets the certificate price p; under the subscription model, she sets of monthly subscription fee f; under the mixed model, she sets both p and f. All the learners then decide whether to purchase the certificate (if there is such an option), subscribe to the course (if there is such an option), or pay nothing. The payments are then made accordingly.

A list of notations is provided in Table 1.

【Table 1】 List of decision variables and parameters

Decision variables	
p	Certificate price
f	Monthly subscription fee
Parameters	
u	Value of certificate
T	The total amount of money paid by a learner
β	The proportion of efficient learners
α_E	The proportion of irrational efficient learners
α_I	The proportion of irrational inefficient learners
t	The actual course complete time of inefficient learners

4、Analysis

(1)、The certificate model

Suppose that the platform chooses the certificate model, i.e., the institution may only offer learners the certificate option. In this case, the institution chooses the certificate price p to maximize her profit. Given p, a learner would purchase the certificate if his utility $U_C = u - p \geq 0$. Note that as the payment amount is independent of the learning time, all learners with

identical willingness-to-pay u will behave the same, regardless of whether one is efficient or inefficient, rational or irrational. Therefore, the number of learners who will purchase the certificate is $1-p$, and the institution's profit maximization problem can be formulated as

$$\max_{p} \ \pi_C(p) = p(1-p). \tag{1}$$

The optimal price and associated profit are summarized in Proposition 1.

Proposition 1. *The optimal certificate price and equilibrium profit under the certificate model are*

$$p_C^* = \frac{1}{2} \qquad \textit{and} \qquad \pi_C^* = \frac{1}{4}. \tag{2}$$

Proposition 1 is a consequence of the very typical price-quantity trade-off. The institution finds it optimal to set the price to a level that best balances the earning per sales and the sales volume. The equilibrium profit $\pi_C^* = \frac{1}{4}$ will be the benchmark of the other two models.

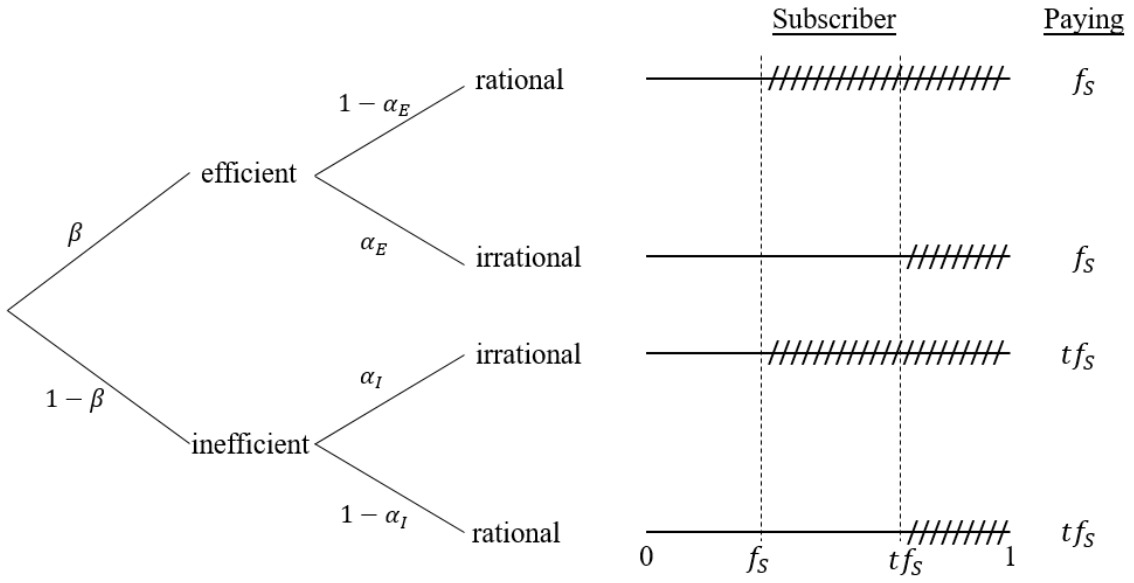
(1)、The subscription model

Suppose that the platform chooses the subscription model, and thus the learners are left with the subscription option only. In this case, the institution chooses the monthly subscription fee f_S to maximize her profit.

Given f_S, now the four types of learners (different in their degrees of efficiency and rationality) make different considerations. On one hand, both a rational efficient learner and an irrational inefficient learner believe that their learning time is 1 and their utility of subscription is $U_S = u - f_S$. Therefore, the proportions of these two types of leaners that will subscribe are both $1-f_S$. However, while the former will only pay f_S to the institution by completing the course in 1 month, the latter will end up paying tf_S. On the other hand, both a rational inefficient learner and an irrational efficient learner believe that their learning time is t and their utility of subscription is $U_S = u - tf_S$. They also pay differently, however: A rational inefficient one really pays tf_S while an irrational efficient one only pays f_S.

Because subscribers now may pay different total amounts (either f_S or tf_S), the institution has two choices in inducing subscription. First, she may set $tf_S < 1$ to induce all four types of learners to subscribe. In this case, the proportions of rational inefficient and irrational efficient leaners that will subscribe are both $1-tf_S$. Alternatively, if the institution set $tf_S \geq 1$, all these learners will not subscribe to the course. While setting $tf_S \geq 1$ may be optimal under the subscription model, this will never be better than the certificate model. To see this, first note that all subscribers pay a single amount just as what happens under the certificate model. However, the number of subscribers, which come from only two types of learners, is lower than the number of purchasers, which come from all four types. As the

objective of this study is the comparison among revenue models, below we will only focus on the case that the monthly subscription fee f_S is lower than $\frac{1}{t}$. The number of subscribers and their paying amounts are visualized in Figure 1.



【Figure 1】Number of subscribers and the paying amounts of the subscription model

By considering the four types of learners' behavior and relative proportions, the institution solves

$$\begin{aligned}\pi_S(f_S) = \beta f_S[(1-\alpha_E)(1-f_S) + \alpha_E(1-tf_S)] \\ +(1-\beta)tf_S[\alpha_I(1-f_S) + (1-\alpha_I)(1-tf_S)].\end{aligned} \tag{3}$$

to maximize her profit. The optimal monthly subscription fee is characterized in Proposition 2.

Proposition 2. *Suppose that* $f_s < \frac{1}{t}$*, the optimal subscription fee and equilibrium profit under the subscription model are*

$$f_S^* = \frac{\beta+t(1-\beta)}{2[\beta(1-\alpha_E+\alpha_E t)+t(1-\beta)(t+\alpha_I-\alpha_I t)]} \text{ and } \pi_S^* = \frac{[\beta+t(1-\beta)]^2}{4[\beta(1-\alpha_E+\alpha_E t)+t(1-\beta)(t+\alpha_I-\alpha_I t)]}. \tag{4}$$

Moreover, we have $f_S^* < \frac{1}{2} = p_C^*$*, and* f_S^* *decreases in* β*, increases in* α_I*, and decreases in* α_E.

According to Proposition 2, we observe that the optimal monthly subscription fee is affected by β, α_I, α_E, and t. Consider β first. When β increases, there are more learners that are efficient. As efficient learners pay less to complete the course, the institution must cut down the price to enlarge the size of subscribers. When α_I increases, there are more learners that consider themselves as efficient. In this case, however, these learners actually pay more than they should. This makes subscription a better idea for the institution to collect revenues from learners. The monthly subscription fee should therefore go up. This also explains why it should go down when α_E increases. Note that $f_S^* < \frac{1}{2} = p_C^*$, which means that the optimal

monthly subscription fee should be lower than the optimal certificate fee. This is intuitive, as otherwise even confident learners find it more expensive under the subscription model. The monthly subscription fee then must be too high.

(3)、The mixed model

The last revenue model that the platform may consider is the mixed model. Under the mixed model, the institution offers the learners both the certificate and subscription options by determining the certificate price p_M and monthly subscription fee f_M. Note that either the certificate model or subscription model is a special case of the mixed model, the mixed model will obviously outperform the other two pure models. For example, the institution may set f_M to be extremely high so that no learner would be willing to subscribe to the course. The mixed model then degenerates to the certificate model. It is still non-obvious, however, whether the mixed model will *strictly* outperform them. One thing for sure is that if the mixed model may strictly outperform the other two models, it must be the case that in equilibrium some learners choose to purchase the certificate while some others choose to subscribe to the course. Otherwise one may mimic the mixed model by one of the two pure models. Therefore, below we will focus on such a differentiating strategy when deriving the optimal pricing decisions for the mixed model.

To differentiate learners, the only possibility is to make the rational efficient and irrational inefficient learners choose subscription and the other two types choose certificate. The former requires $f_S \leq p_C$, so that one who believes that he is efficient will choose subscription and try to complete the course in one month. The latter requires $p_C \leq tf_S$, which makes those unconfident learners feel that purchasing a certificate is better than paying the subscription fee for t months. These two constraints will be imposed on the institution's profit maximization problem.

Given the certificate price p_M and monthly subscription fee f_M, each learner would compare the three options (purchasing the certificate, subscribing to the course, and paying nothing) and choose the one with the highest utility. Consider a rational efficient learner first. As he is efficient, he will prefer to subscribe and eventually pay f_M. Similarly, an irrational inefficient learner will also subscribe. However, he will end up with paying tf_M if he really subscribe. On the contrary, rational inefficient and irrational efficient learners will all prefer the certificate option by paying p_M. Learners whose willingness-to-pay are high enough will subscribe to the course or purchase the certificate according to their preferences. Figure 2 depicts the numbers of subscribers and purchasers and their paying amounts.

Collectively, the institution's profit maximization problem is

$$\begin{aligned} \max_{p_M, f_M} \quad & \beta[(1-\alpha_E)(1-f_M)f_M + \alpha_E(1-p_M)p_M] \\ & +(1-\beta)[\alpha_I(1-f_M)tf_M + (1-\alpha_I)(1-p_M)p_M] \\ \text{s.t.} \quad & f_M \leq p_M \leq tf_M. \end{aligned} \tag{5}$$

We summarize the optimal certificate price and monthly subscription fee in the next proposition.

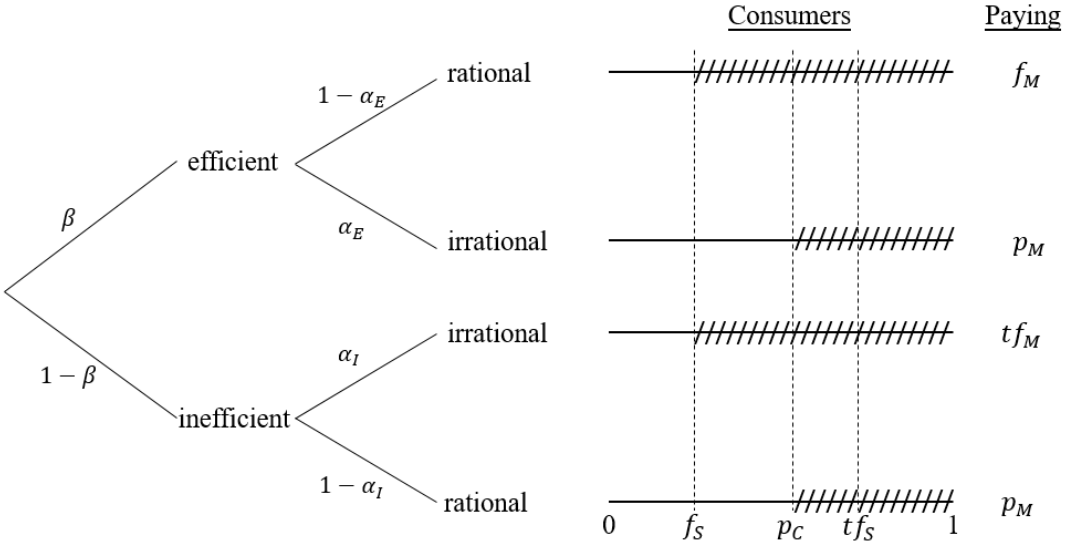
Interestingly, the two amounts are identical.

Proposition 3. *The optimal certificate price, optimal monthly subscription fee, and equilibrium profit under the mixed model are*

$$f_M^* = \frac{1}{2}, \qquad p_M^* = \frac{1}{2}, \qquad \textit{and} \qquad \pi_M^* = \frac{1}{4}[1 + \alpha_I(1-\beta)(t-1)]. \tag{6}$$

Under the mixed model, the optimal monthly subscription fee and certificate price are "identical." Note that this does not mean subscribing to the course and purchasing the certificate are equivalent. There are still some learners, those irrational inefficient learners, who will end up paying $tf_M^* = \frac{t}{2}$, a price higher than the certificate price. They think they may pay just f_M^*, but they are wrong. The subscription model is needed to extract as much surplus as possible from them. When α_I increases, there are more rational inefficient learners, and naturally the institution may earn more.

Somewhat surprisingly, however, the proportion of efficient learners that are irrational, α_E, does not affect the institution's profit. To understand this, note that under the optimal pricing plan, an irrational efficient learner pays p_M to purchase a certificate while a rational efficient learner pays f_M by subscribing to the course. The irrational one does not pay less! In other words, the institution finds it optimal to equalize the monthly subscription fee and the certificate price. This eliminates the potential detriment brought by irrational efficient learners.



【Figure 2】Number of consumers and the paying amounts of the mixed model

(4)、Comparison

Now we are ready to compare the three revenue models regarding their profitability. By comparing π_C^*, π_S^*, and π_M^* derived in Propositions 1, 2, and 3, we obtain the next proposition.

Proposition 4. *Regarding profitability:*

(a) *If all inefficient learners are rational, the mixed model and the certificate model are equally good. Moreover, they both strictly outperform the subscription model:* $\pi_M^* = \pi_C^* > \pi_S^*$ *if* $\alpha_I = 0$.

(b) *If some inefficient learners are irrational, the mixed model strictly outperforms the certificate model, which strictly outperforms the subscription model:* $\pi_M^* > \pi_C^* > \pi_S^*$ *if* $\alpha_I > 0$.

(c) *Whether there exist irrational efficient learners does not matter.*

Proposition provides several insights to us. First, if all learners are rational (more precisely, if all inefficient learners are rational), there is no need for the institution to introduce the subscription model to learners. The pure certificate model is sufficient to maximize the institution's profit. As the pure certificate model does not require institutions and MOOC platforms to prevent unpaid learners viewing course materials, it is arguably the least costly to offer. The certificate model is thus optimal.

If this is the case, why did Coursera and other major MOOC platforms start to introduce the subscription option? According to our study, learners' bounded rationality on their efficiency may be one reason. As long as there is a learner overestimate his efficiency (and thus underestimate the learning time), he will end up paying "too much." Institutions as well as MOOC platforms then benefit from the existence of these learners with bounded rationality.

Finally, whether some learners underestimate their efficiency does not affect the optimality of the mixed model. Note that it does affect the profit under the subscription model: If more learners underestimate their efficiency and pay less than they expect, the profit under the subscription model will decrease. Then why the detriment disappears under the mixed model? According to Proposition 4, we find that all irrational efficient learners choose to purchase a certificate. This implies that, by adding the certificate option into the pure subscription model to create the mixed model, the institution may carefully design the prices and induce irrational efficient learner to prefer certificate to subscription. They then all pay the amount they "should" pay, and their existence does not hurt the institution.

5、Conclusion

In this paper, we present a game-theoretic model featuring learner's rationality to study MOOCs pricing strategy. We introduce three revenue models including the certificate model, the subscription model, and the mixed model. In general, if learners all are fully rational, the platform never needs to consider the subscription option. However, the bounded rationality of learners makes the subscription option more attractive. Though a pure subscription model is never optimal, the mixed model combining subscription and certificate is the most favorable. Interestingly, while the existence of irrational inefficient learners benefits the institution and

platform, it does not matter whether irrational efficient learners exist or not. The potential detriment brought by the irrational efficient learners can somehow be eliminated by the optimal mixed pricing strategy.

There are several possible ways to extend this study. For instance, in practice learners may improve their efficiency by exerting efforts. Including this will broaden our analysis about the subscription model. Moreover, we assume that a learner is either fully rational or completely irrational in this study. If the heterogeneity on irrationality extends to be presented like a spectrum between full rationality and full irrationality, the optimal pricing strategy may also change. These extensions of our study call for future investigation.

A、 Appendix

Proof of Proposition 1. By completing the problem of $\pi_C(p_C)$, we get $\pi_C(p_C) = p_C(1 - p_C)$. The first- and second-order derivatives of $\pi_C(p_C)$ with respect to p_C are

$$\frac{\partial \pi_C(p_C)}{\partial p_C} = 1 - 2p_C \quad and \quad \frac{\partial^2 \pi_C(p_C)}{\partial p_C^2} = -2,$$

Because $\frac{\partial^2 \pi_C(p_C)}{\partial p_C^2} < 0$, the function is concave. For the concavity, the optimal solution must satisfy $\frac{\partial \pi_C(p_C^*)}{\partial p_C} = 0$, i.e., $p_C^* = \frac{1}{2}$. We plug in this into π_c^* , and we can obtain $\pi_c^* = \frac{1}{4}$. ■

Proof of Proposition 2. The first- and second-order derivatives of $\pi_S(f_S)$ with respect to f_S are

$$\frac{\partial \pi_S(f_S)}{\partial f_S} = \beta[(1 - \alpha_E)(1 - 2f_S) + \alpha_E(1 - 2tf_S)] + (1 - \beta)t[\alpha_I(1 - 2f_S) + (1 - \alpha_I)(1 - 2tf_S)]$$

and

$$\frac{\partial^2 \pi_S(f_S)}{\partial f_S^2} = -2\{\beta[1 + (t - 1)\alpha_E] + (1 - \beta)t[t - (t - 1)\alpha_I]\}.$$

Because $\frac{\partial^2 \pi_S(f_S)}{\partial f_S^2} < 0$, the function is also concave. For the concavity, the optimal solution must satisfy $\frac{\partial \pi_S(f_S^*)}{\partial f_S} = 0$, i.e., $f_S^* = \frac{\beta + t(1-\beta)}{2[\beta(1-(t-1)\alpha_E) + t(1-\beta)(t-(t-1)\alpha_I)]}$. The optimal profit under subscription model π_S^* is therefore $\pi_S(f_S^*)$. Note that

$$f_S^* = \frac{\beta + t(1 - \beta)}{2[\beta + t(1 - \beta)] + 2[\beta(t - 1)\alpha_E + t(1 - \beta)(t - 1)(1 - \alpha_I)]} < \frac{1}{2}.$$

f_S^* may also be shown to be no greater than $\frac{1}{t}$ as long as $t < 2$. Finally, the derivatives of f_S^* with respect to β, α_I, *and* α_E may be easily shown to be negative, positive, and negative, respectively. ■

Proof of Proposition 3. By reorganizing the profit maximization problem of the mixed model, the problem become

$$\begin{aligned} \max_{p_M, f_M} \quad & f_M(1-f_M)[\beta(1-\alpha_E)+t(1-\beta)\alpha_I]+p_M(1-p_M)[\beta\alpha_E+(1-\beta)(1-\alpha_I)] \\ \text{s.t.} \quad & f_M \le p_M \le t f_M, \end{aligned}$$

If we ignore the constraints, both the optimal price f_M^* and p_M^* are obviously $\frac{1}{2}$. As they satisfies all the constraints, they are indeed optimal. We then substitute the two optimal price into the problem, and obtain the maximum profit of the mixed model $\pi_M^* = \frac{1}{4}[1+\alpha_I(1-\beta)(t-1)]$. ■

Proof of Proposition 4.

To show that the certificate model is out performance the subscription model, we simply subtract π_S^* from π_C^*, and get $\pi_C^* - \pi_S^* = \frac{-1}{4}[\beta(\beta-1)(1-t)^2+\alpha_E(1-t)\beta+\alpha_I t(1-t)(\beta-1)]$. Note that $\pi_C^* - \pi_S^*$ decreases in α_I and is that minimized at $\alpha_I = 1$. As $\pi_C^* - \pi_S^*$ can be shown to be positive when $\alpha_I = 1$, it is confirmed that $\pi_C^* > \pi_S^*$ in all cases.

As for the profitability of the mixed model and the certificate model, recall that $\alpha_I = 0$ represents all inefficient learners are rational. When this happenes, the maximum profit of the mixed model becomes $\frac{1}{4}$, which is the same as the maximum profit of the certificate model, while when $\alpha_I > 0$, π_M^* is obviously greater than π_C^*.

Recall that α_E represents the proportion of irrational in efficient learners. When $\alpha_E = 0$, $\pi_C^* - \pi_S^*$ still positive, and apparently α_E has no influence on π_M^*. Hence it is proved that the existence of α_E does not affect the outcome. ■

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